The hard to soft Pomeron transition in small x DIS data using optimal renormalization

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Abstract

We show that it is possible to describe the effective Pomeron intercept, determined from the HERA Deep Inelastic Scattering data at small values of Bjorken x, using next-to-leading order BFKL evolution together with collinear improvements. To obtain a good description over the whole range of Q^2 we use a non-Abelian physical renormalization scheme with BLM optimal scale, combined with a parametrization of the running coupling in the infrared region.

1 Introduction & theoretical approach

The description of Deep Inelastic Scattering (DIS) data for the structure function F_2 in different regions of Bjorken x and virtuality of the photon Q^2 is one of the classical problems in perturbative QCD. The literature on the subject is very large (see, *e.g.*, the reviews in Ref. [1]). In the present letter we are interested in regions with low values of x and revisit the theoretical approach to the problem using the next-to-leading order (NLO) [2] BFKL [3] equation together with collinear improvements. We find that, in order to get a good description over the full range of Q^2, we can use optimal renormalization schemes. In this work we focus on indicating which are the most important theoretical aspects which drive the bulk of our results. It is possible to introduce subleading refinements in our calculation which make our predictions even closer to the data and will be presented elsewhere.

Let us first review some well-known formulas for DIS in order to set the ground for our treatment of the small x resummation. In DIS the cross section is written in terms of the structure functions F_2 and F_L in the form

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left\{\left[1+(1-y)^2\right]F_2(x,Q^2) - y^2 F_L(x,Q^2)\right\}, \qquad (1)$$

where x and y are the dimensionless Bjorken variables, Q^2 the photon's virtuality and α the electromagnetic constant. More explicitly, for the structure functions, in terms of transverse and longitudinal polarizations of the photon, we have

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha}[\sigma_T(x,Q^2)+\sigma_L(x,Q^2)],\ F_L(x,Q^2) = \frac{Q^2}{4\pi^2\alpha}\sigma_L(x,Q^2), \quad (2)$$

where $\sigma_{L,T}$ is the cross-section for the scattering of a transverse (longitudinal) polarized virtual photon on the proton. At large center-of-mass energy $\sqrt{s}$, which corresponds to the small $x \simeq Q^2/s$ limit, high energy factorization makes it possible to write the structure functions F_I, $I = 2, L$ in the form

$$F_I(x,Q^2) = \frac{1}{(2\pi)^4}\int\frac{d^2\mathbf{q}_\perp}{q^2}\int\frac{d^2\mathbf{p}_\perp}{p^2}\Phi_I\left(q,Q^2\right)\Phi_P\left(p,Q_0^2\right)\mathcal{F}\left(s,q,p\right), \ (3)$$

where all the integrations take place in the two-dimensional transverse momenta space with $q = \sqrt{\mathbf{q}_\perp^2}$. The proton ($\Phi_P$) and photon ($\Phi_I$) impact factors are functions which are dominated by $\mathcal{O}(Q_0)$ and $\mathcal{O}(Q)$ transverse scales, respectively. Note that the dependence of Φ_I on the photon virtuality can be calculated in perturbation theory. This is not the case for Φ_P whose dependence on the non-perturbative scale $Q_0 \simeq \Lambda_{\rm QCD}$ can only be modeled.

If Q^2 was a scale similar to Q_0^2 then the gluon Green's function $\mathcal{F}$, which corresponds to the solution of the BFKL equation, would be written as

$$\mathcal{F}(s,q,p) = \frac{1}{2\pi q\,p}\int\frac{d\omega}{2\pi i}\int\frac{d\gamma}{2\pi i}\left(\frac{q^2}{p^2}\right)^{\gamma-\frac{1}{2}}\left(\frac{s}{q\,p}\right)^{\omega}\frac{1}{\omega-\bar{\alpha}_s\chi_0\left(\gamma\right)}, \quad (4)$$

with $\bar{\alpha}_s = \alpha_s N_c/\pi$ and $\chi_0(\gamma) = 2\psi(1)-\psi(\gamma)-\psi(1-\gamma)$ in a leading order (LO) approximation, which resums $\bar{\alpha}_s^n \log^n s$ terms to all-orders in the strong coupling. $\psi(\gamma)$ is the logarithmic derivative of the Euler Gamma function. In DIS, however, $Q^2 \gg Q_0^2$ and this expression should be written in a form consistent with the resummation of $\bar{\alpha}_s \log{(1/x)}$ contributions:

$$\mathcal{F}(s,q,p) = \frac{1}{2\pi q^2}\int\frac{d\omega}{2\pi i}\int\frac{d\gamma}{2\pi i}\left(\frac{q^2}{p^2}\right)^{\gamma}\left(\frac{s}{q^2}\right)^{\omega}\frac{1}{\omega-\bar{\alpha}_s\chi_0\left(\gamma-\frac{\omega}{2}\right)}. \quad (5)$$

It is well-known that the zeros of the denominator in the integrand generate in the limits $\gamma \to 0, 1$ all-orders terms not compatible with DGLAP evolution [4, 5]. The first of these pieces ($\mathcal{O}(\alpha_s^2)$) is removed when the NLO correction to the BFKL kernel is taken into account but not the higher order ones, which remain and are numerically important. A scheme to eliminate these spurious contributions [4], in a nutshell, consists of using a modified BFKL kernel in Eq. (4) where we essentially introduce the change $\chi_0(\gamma) \to \chi_0(\gamma + \omega/2)$.

Let us present now in a precise manner our procedure to include the NLO corrections and collinear improvements. The NLO expansion of the BFKL kernel in terms of poles at $\gamma = 0, 1$ reads

$$\begin{aligned}
\omega &= \bar{\alpha}_s \chi_0(\gamma - \frac{\omega}{2}) + \bar{\alpha}_s^2 \chi_1(\gamma) \\
&= \bar{\alpha}_s \chi_0(\gamma) + \bar{\alpha}_s^2 \chi_1(\gamma) - \frac{1}{2}\bar{\alpha}_s^2 {\chi_0}'(\gamma)\chi_0(\gamma) + \mathcal{O}(\bar{\alpha}_s^3) \\
&\simeq \frac{\bar{\alpha}_s}{\gamma} + \bar{\alpha}_s^2 \left(\frac{a}{\gamma} + \frac{b}{\gamma^2} - \frac{1}{2\gamma^3}\right) + \frac{\bar{\alpha}_s}{1-\gamma} + \frac{\bar{\alpha}_s^2}{2\gamma^3} - \frac{\bar{\alpha}_s^2}{2(1-\gamma)^3} \\
&+ \bar{\alpha}_s^2 \left[\frac{a}{1-\gamma} + \frac{b}{(1-\gamma)^2} - \frac{1}{2(1-\gamma)^3}\right] + \mathcal{O}(\bar{\alpha}_s^3),
\end{aligned} \tag{6}$$

where ${\chi_0}'(\gamma) = \psi'(1-\gamma) - \psi'(\gamma)$. Now, as we have explained before, we resum in the Regge region ($Q^2 \simeq Q_0^2$) collinear logarithms by introducing a shift of the general form [4, 5]

$$\omega = \bar{\alpha}_s(1 + A\bar{\alpha}_s)\left[2\psi(1) - \psi\left(\gamma + \frac{\omega}{2} + B\bar{\alpha}_s\right) - \psi\left(1 - \gamma + \frac{\omega}{2} + B\bar{\alpha}_s\right)\right]. \tag{7}$$

When translated into the DIS limit ($Q^2 \gg Q_0^2$) this expression is to be replaced by

$$\begin{aligned}
\omega &= \bar{\alpha}_s(1 + A\bar{\alpha}_s)\left[2\psi(1) - \psi\left(\gamma + B\bar{\alpha}_s\right) - \psi\left(1 - \gamma + \omega + B\bar{\alpha}_s\right)\right] \\
&= \bar{\alpha}_s(1 + A\bar{\alpha}_s)\sum_{m=0}^{\infty}\left(\frac{1}{\gamma + m + B\bar{\alpha}_s} + \frac{1}{1 - \gamma + m + \omega + B\bar{\alpha}_s} - \frac{2}{m+1}\right).
\end{aligned} \tag{8}$$

It is possible to get a very good approximation to the solution of this equation (certainly within the uncertainty of the resummation scheme) by breaking its transcendentality and solving it pole by pole and summing up the different solutions. This procedure was proposed in Ref. [5]. In terms of (anti-)collinear

poles we obtain

$$
\begin{aligned}
\omega &= \sum_{m=0}^{\infty}\left\{\bar{\alpha}_s(1+A\bar{\alpha}_s)\left(\frac{1}{\gamma+m+B\bar{\alpha}_s}-\frac{2}{m+1}\right)\right.\\
&+ \left.\frac{1}{2}\left(\gamma-1-m-B\bar{\alpha}_s+\sqrt{(\gamma-1-m-B\bar{\alpha}_s)^2+4\bar{\alpha}_s(1+A\bar{\alpha}_s)}\right)\right\}\\
&= \sum_{m=0}^{\infty}\left\{\bar{\alpha}_s\left(\frac{1}{\gamma+m}+\frac{1}{1-\gamma+m}-\frac{2}{m+1}\right)\right.\\
&+ \bar{\alpha}_s^2\left(\frac{A}{\gamma+m}+\frac{A}{1-\gamma+m}-\frac{B}{(\gamma+m)^2}-\frac{B}{(1-\gamma+m)^2}\right.\\
&- \left.\left.\frac{1}{(1+m-\gamma)^3}-\frac{2A}{m+1}\right)\right\}+\mathcal{O}(\bar{\alpha}_s^3).
\end{aligned}
\tag{9}
$$

In order to match the NLO poles in Eq. (6) we need to fix $A=a$ and $B=-b$. Keeping the LO and NLO kernels unmodified and introducing only higher orders corrections, our collinearly improved BFKL kernel then simply reads

$$
\chi(\gamma) = \bar{\alpha}_s\chi_0(\gamma)+\bar{\alpha}_s^2\chi_1(\gamma)-\frac{1}{2}\bar{\alpha}_s^2{\chi_0}'(\gamma)\chi_0(\gamma)+\chi_{\mathrm{RG}}(\bar{\alpha}_s,\gamma,a,b), \tag{10}
$$

with

$$
\begin{aligned}
\chi_{\mathrm{RG}}(\bar{\alpha}_s,\gamma,a,b) &= \bar{\alpha}_s(1+a\bar{\alpha}_s)\left(\psi(\gamma)-\psi(\gamma-b\bar{\alpha}_s)\right)\\
&-\frac{\bar{\alpha}_s^2}{2}\psi''(1-\gamma)-b\bar{\alpha}_s^2\frac{\pi^2}{\sin^2(\pi\gamma)}+\frac{1}{2}\sum_{m=0}^{\infty}\left(\gamma-1-m+b\bar{\alpha}_s\right.\\
&\left.-\frac{2\bar{\alpha}_s(1+a\bar{\alpha}_s)}{1-\gamma+m}+\sqrt{(\gamma-1-m+b\bar{\alpha}_s)^2+4\bar{\alpha}_s(1+a\bar{\alpha}_s)}\right).
\end{aligned}
\tag{11}
$$

For the NLO kernel,

$$
\begin{aligned}
\chi_1(\gamma) &= \mathcal{S}\chi_0(\gamma)-\frac{\beta_0}{8N_c}\chi_0^2(\gamma)+\frac{\Psi''(\gamma)+\Psi''(1-\gamma)-\phi(\gamma)-\phi(1-\gamma)}{4}\\
&-\frac{\pi^2\cos(\pi\gamma)}{4\sin^2(\pi\gamma)(1-2\gamma)}\left[3+\left(1+\frac{n_f}{N_c^3}\right)\frac{2+3\gamma(1-\gamma)}{(3-2\gamma)(1+2\gamma)}\right]+\frac{3}{2}\zeta(3),
\end{aligned}
\tag{12}
$$

with $\mathcal{S}=(4-\pi^2+5\beta_0/N_c)/12$, $\beta_0=\left(\frac{11}{3}N_c-\frac{2}{3}n_f\right)$ and

$$
\begin{aligned}
\phi(\gamma)+\phi(1-\gamma) &=\\
&\sum_{m=0}^{\infty}\left(\frac{1}{\gamma+m}+\frac{1}{1-\gamma+m}\right)\left[\Psi'\left(1+\frac{m}{2}\right)-\Psi'\left(\frac{1+m}{2}\right)\right],
\end{aligned}
\tag{13}
$$

we obtain for the coefficients

$$a \;=\; \frac{5}{12}\frac{\beta_0}{N_c} - \frac{13}{36}\frac{n_f}{N_c^3} - \frac{55}{36}, \qquad b \;=\; -\frac{1}{8}\frac{\beta_0}{N_c} - \frac{n_f}{6N_c^3} - \frac{11}{12}. \tag{14}$$

As we have already indicated, the non-perturbative proton impact factor has to be modeled. We take the functional form

$$\Phi_P\left(p, Q_0^2\right) \;=\; \mathcal{C}\left(\frac{p^2}{Q_0^2}\right)^{\delta} e^{-\frac{p^2}{Q_0^2}}, \tag{15}$$

which introduces three independent free parameters and has a maximum at $p^2 = \delta\, Q_0^2$. Its representation in γ space reads

$$\int \frac{d^2p}{p^2}\Phi_P\left(p, Q_0^2\right)(p^2)^{-\gamma} \;=\; \pi\,\mathcal{C}\,\Gamma(\delta-\gamma)(Q_0^2)^{-\gamma}. \tag{16}$$

In the present work we choose to keep the treatment of the impact factors as simple as possible in order to focus on the behaviour of the gluon Green's function. Having this philosophy in mind, we work with the LO photon impact factor which can be written in the form (directly in $\nu = i(1/2-\gamma)$ space)

$$\int \frac{d^2q}{q^2}\Phi_I\left(q, Q^2\right)\left(\frac{q^2}{Q^2}\right)^{\gamma-1} \;=\; \alpha\,\bar{\alpha}_s\,\pi^4 \sum_{q=1}^{n_f} e_q^2 \frac{\Omega_I(\nu)}{\nu+\nu^3}\,\mathrm{sech}(\pi\nu)\,\tanh(\pi\nu)\,, \tag{17}$$

where $\Omega_2 = (11+12\nu^2)/8$ and $\Omega_L = \nu^2 + 1/4$.

So far we have not included in the NLO kernel those terms breaking scale invariance, directly linked to the running of the coupling. They appear as a differential operator in ν space which acts on the impact factors (for a similar analysis see Ref. [6]). Exponentiating only the scale invariant LO and NLO terms in the kernel, the structure functions can be written as

$$\begin{aligned} F_I(x,Q^2) \;&=\; \mathcal{D}\int_{-\infty}^{\infty} d\nu\, x^{-\chi\left(\frac{1}{2}+i\nu\right)} c_I(\nu) c_P(\nu) \Bigg\{ 1 \\ &+\bar{\alpha}_s^2 \log\left(\frac{1}{x}\right)\frac{\beta_0}{8N_c}\chi_0\left(\frac{1}{2}+i\nu\right)\left[\log\left(\mu^4\right) + i\frac{d}{d\nu}\log\left(\frac{c_I(\nu)}{c_P(\nu)}\right)\right]\Bigg\}, \end{aligned} \tag{18}$$

where we have gathered different constants in $\mathcal{D}$ and μ denotes the renormalization scale at which the QCD coupling is evaluated. Since

$$c_I(\nu) \;=\; (Q^2)^{\frac{1}{2}+i\nu}\frac{\Omega_I(\nu)}{\nu+\nu^3}\,\mathrm{sech}(\pi\nu)\,\tanh(\pi\nu), \tag{19}$$

$$c_P(\nu) \;=\; \Gamma\left(\delta - \frac{1}{2} - i\nu\right)(Q_0^2)^{-\frac{1}{2}-i\nu}, \tag{20}$$

we can write our final expression in the form

$$
\begin{aligned}
F_I(x,Q^2) &= \mathcal{D}\int_{-\infty}^{\infty} d\nu\, x^{-\chi\left(\frac{1}{2}+i\nu\right)} c_I(\nu) c_P(\nu) \Bigg\{ 1 \\
&\quad + \bar{\alpha}_s^2 \log\left(\frac{1}{x}\right) \frac{\beta_0}{8N_c} \chi_0\left(\frac{1}{2}+i\nu\right) \left[-\log\left(\frac{Q^2 Q_0^2}{\mu^4}\right) - \psi\left(\delta - \frac{1}{2} - i\nu\right) \right. \\
&\quad \left. + i\left(\pi\coth(\pi\nu) - 2\pi\tanh(\pi\nu) - M_I(\nu) \right) \right] \Bigg\}, \qquad (21)
\end{aligned}
$$

where

$$
M_2(\nu) = \frac{11+21\nu^2+12\nu^4}{\nu(1+\nu^2)(11+12\nu^2)}, \quad M_L(\nu) = \frac{1-\nu^2+4\nu^4}{\nu(1+5\nu^2+4\nu^4)}. \qquad (22)
$$

Although we have included all the ingredients needed to calculate F_L, we leave a comparison to experimental data for this observable to future work and focus in the following on F_2.

2 Running coupling & optimal renormalization

Although there is some freedom in the treatment of the running of the coupling, it is natural to remove the μ dependent logarithm in the second line of Eq. (21) making the replacement

$$
\bar{\alpha}_s - \bar{\alpha}_s^2 \frac{\beta_0}{8N_c} \log\left(\frac{Q^2 Q_0^2}{\mu^4}\right) \longrightarrow \bar{\alpha}_s\left(QQ_0\right), \qquad (23)
$$

and use this resummed coupling throughout our calculations. We are interested in the comparison of our approach with DIS data in the small x region. In this letter we focus on the description of the Q^2 dependence of the well-known effective intercept $\lambda(Q^2)$, which can be obtained from experimental DIS data in the region $x < 10^{-2}$ through a parametrization of the structure function of the form $F_2(x,Q^2) = c(Q^2)x^{-\lambda(Q^2)}$. The intercept $\lambda(Q^2)$ is $\mathcal{O}(0.3)$ at large values of Q^2 and $\mathcal{O}(0.1)$ at low values, closer to the confinement region. This can be qualitatively interpreted as a smooth transition from hard to soft Pomeron exchange. When trying to describe these data with our approach we have found that it is rather difficult to get good agreement over the full range of $1\,\mathrm{GeV}^2 < Q^2 < 300\,\mathrm{GeV}^2$. Somehow it is needed to introduce some new ideas related to the infrared region. A recent very interesting possibility is

that proposed by Kowalski, Lipatov, Ross and Watt in Ref. [7]. Alternatively, we have found that moving from the $\overline{\text{MS}}$ scheme to renormalization schemes inspired by the existence of a possible infrared fixed point significantly helps in generating a natural fit for $\lambda(Q^2)$, in the sense of having sensible values for the two free parameters in our calculation which affect this observable: δ and Q_0 in the proton impact factor. Here we are guided by having a proton impact factor which should be dominated by the infrared region. In the following we provide some details of our findings.

The first evaluation of the BFKL Pomeron intercept in non-Abelian physical renormalization schemes using the Brodsky-Lepage-Mackenzie (BLM) optimal scale setting [8] was performed in Ref. [9] in the context of virtual photon-photon scattering. We will use the same procedure in our calculation. The pieces of the BFKL kernel at NLO proportional to β_0 are isolated and absorbed in a new definition of the running coupling in such a way that all vacuum polarization effects from the β_0 function are resummed, *i.e.*,

$$\tilde{\alpha}_s\left(QQ_0,\gamma\right) = \frac{4N_c}{\beta_0\left[\log\left(\frac{QQ_0}{\Lambda^2}\right)+\frac{1}{2}\chi_0(\gamma)-\frac{5}{3}+2\left(1+\frac{2}{3}Y\right)\right]}, \tag{24}$$

where we are using the momentum space (MOM) physical renormalization scheme based on a symmetric triple gluon vertex [10] with $Y \simeq 2.343907$ and gauge parameter $\xi = 3$ (our results are very weakly dependent on this choice). This scheme is more suited to the BFKL context since there are large non-Abelian contributions to the kernel. The replacements we need in our kernel in order to introduce this new scheme are $\bar{\alpha}_s\left(QQ_0\right) \to \tilde{\alpha}_s\left(QQ_0\right)$ in Eq.(23) and $\chi_1(\gamma) \to \tilde{\chi}_1(\gamma)$ in Eq. (12) together with the corresponding adjustments for the coefficients $a, b \to \tilde{a}, \tilde{b}$ which enter Eq. (11). They read

$$\begin{aligned}
\tilde{\chi}_1(\gamma) &= \tilde{\mathcal{S}}\chi_0(\gamma)+\frac{3}{2}\zeta(3)+\frac{\Psi''(\gamma)+\Psi''(1-\gamma)-\phi(\gamma)-\phi(1-\gamma)}{4}\\
&- \frac{\pi^2\cos(\pi\gamma)}{4\sin^2(\pi\gamma)(1-2\gamma)}\left[3+\left(1+\frac{n_f}{N_c^3}\right)\frac{2+3\gamma(1-\gamma)}{(3-2\gamma)(1+2\gamma)}\right]\\
&+ \frac{1}{8}\left[\frac{3}{2}(Y-1)\xi+\left(1-\frac{Y}{3}\right)\xi^2+\frac{17Y}{2}-\frac{\xi^3}{6}\right]\chi_0(\gamma), \qquad (25)\\
\tilde{a} &= -\frac{13}{36}\frac{n_f}{N_c^3}-\frac{55}{36}+\frac{3Y-3}{16}\xi+\frac{3-Y}{24}\xi^2-\frac{1}{48}\xi^3+\frac{17}{16}Y \qquad (26)\\
\tilde{b} &= -\frac{n_f}{6N_c^3}-\frac{11}{12}, \qquad (27)
\end{aligned}$$

where $\tilde{\mathcal{S}} = (4-\pi^2)/12$.

In order to access regions with $Q^2 \simeq 1\,\mathrm{GeV}^2$, we use a simple parametrization of the running coupling introduced by Webber in Ref. [11]:

$$\alpha_s\left(\mu^2\right) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2}{\Lambda^2}} + f\left(\frac{\mu^2}{\Lambda^2}\right), \quad f\left(\frac{\mu^2}{\Lambda^2}\right) = \frac{4\pi}{\beta_0} \frac{125\left(1+4\frac{\mu^2}{\Lambda^2}\right)}{\left(1-\frac{\mu^2}{\Lambda^2}\right)\left(4+\frac{\mu^2}{\Lambda^2}\right)^4}. \tag{28}$$

At low scales it is consistent with global data of power corrections to perturbative observables, while for larger values it coincides with the conventional perturbative running coupling constant with Landau pole as shown in Fig. 1.

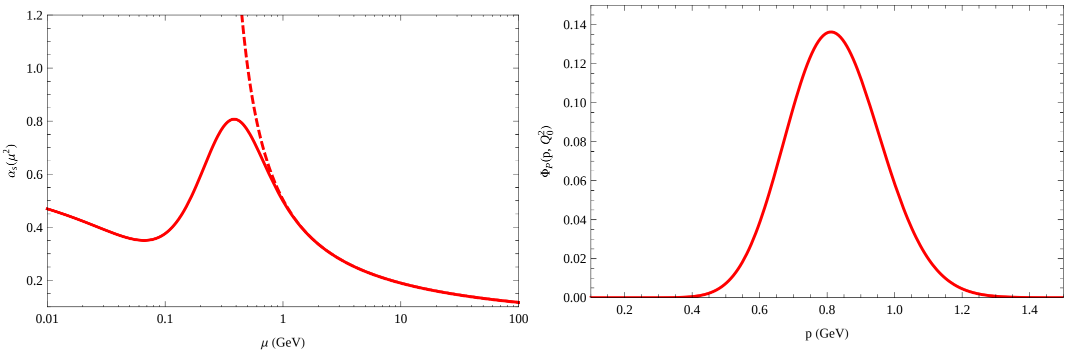

Figure 1: Left: model for the running coupling with freezing in the infrared (solid line) and leading order running coupling with Landau pole (dashed line) for $n_f = 3$ and $\Lambda = 0.25$ GeV. Right: proton impact factor in momentum space with $\mathcal{C} = 1/\Gamma(1+\delta)$ and δ, Q_0 with the values used for the comparison to DIS data.

The final expression used in the numerical analysis is then given by

$$\hat{\alpha}_s\left(QQ_0,\gamma\right) = \tilde{\alpha}_s\left(QQ_0,\gamma\right) + \frac{N_c}{\pi} f\left(\frac{QQ_0}{\Lambda^2}\right), \tag{29}$$

which replaces Eq. (24) in all expressions. In a future publication we will compare the scheme here presented to other physical renormalization schemes. For simplicity we have not introduced a complete treatment of quark thresholds in the results of this letter, but we have checked that they have a very small effect. Let us stress that in our numerical results we do not use any saddle point approximation and perform the numerical integrations exactly.

3 Comparison to DIS data & scope

To obtain our theoretical results we have calculated the logarithmic derivative $\frac{d\log F_2}{d\log(1/x)}$ using Eq. (21) with the modifications described in Section 2. For the comparison with DIS data we chose the values $Q_0 = 0.28\,\mathrm{GeV}$ and $\delta = 8.4$ while the dependence on the overall normalization factor $\mathcal{C}$ cancels for our observable. The QCD running coupling constant is evaluated for $n_f = 4$ and $\Lambda = 0.21\,\mathrm{GeV}$, corresponding to a $\overline{\mathrm{MS}}$ coupling of $\alpha_s^{\overline{\mathrm{MS}}}(M_Z^2) = 0.12$. The result is shown in Fig. 2. The experimental input has been derived from the combined analysis performed by H1 and ZEUS in Ref. [12] with $x < 10^{-2}$. In the results indicated with "Real cuts" we have calculated the effective intercept for F_2 at a fixed Q^2, averaging its values in a sample of x space consistent with the actual experimental cuts in x. To generate the continuous line with label "Smooth cuts" we have used as boundaries in x space those shown in Fig. 3, which correspond to an interpolation of the real experimental boundaries. Note that the difference between both approaches is very small.

We would like to stress the accurate description of the combined HERA data in our approach, in particular at very low values of Q^2. It is noteworthy that the values of Q_0 and δ indicate that our proton impact factor (see the plot at the right in Fig. 1) safely lies within the non-perturbative region since it has its maximum at $\sim 0.81\,\mathrm{GeV}$. In the present letter our intention is to emphasize that, in order to reach the low Q^2 region with a collinearly improved BFKL equation we needed to call for optimal renormalization and use some model with a frozen coupling in the infrared.

It is possible to improve the quality of our fit by introducing subleading contributions such as threshold effects in the running of the coupling, heavy quark masses and higher order corrections to the photon impact factor which became recently available [13]. We leave these, together with a comparison to data not averaged over x, for a more extensive study, which will include an investigation of F_L, to be presented elsewhere.

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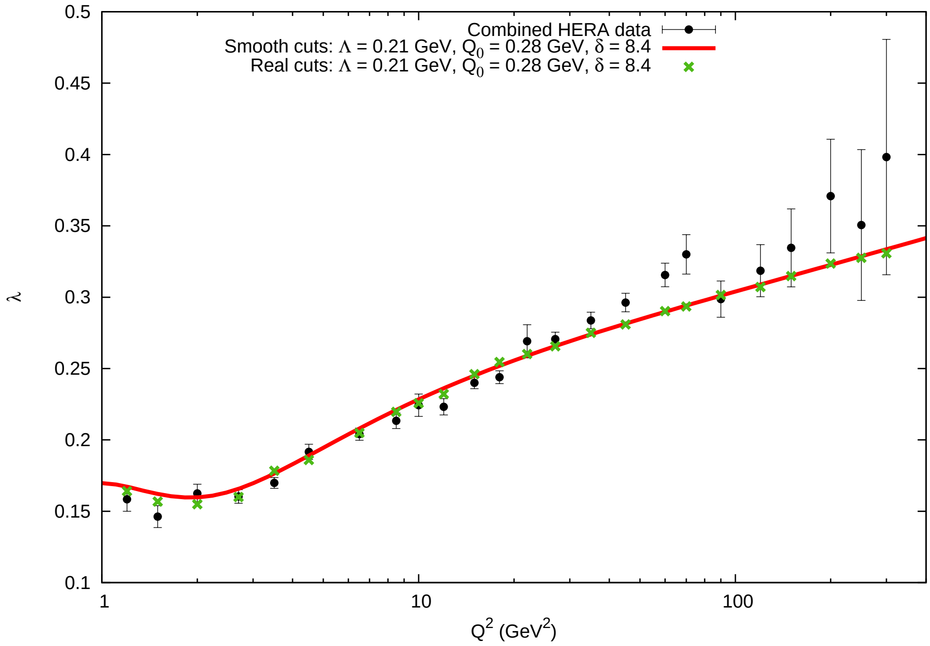

Figure 2: Comparison of our prediction with experimental data.

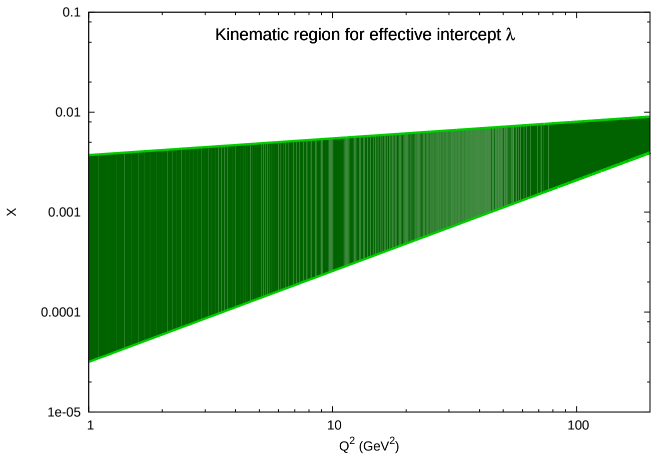

Figure 3: Smooth cuts in x used for the effective intercept of F_2.

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